ZENITH STUDY HUB

SPPU • THIRD YEAR ROBOTICS & AUTOMATION • 2024 PATTERN

Robot Kinematics & Dynamics

UNIT I | Introduction to Robotics and Kinematics

PCC-303-ROA • Semester V • 08 teaching hours

Expanded Concept + Exam Guide

DETAILED CONCEPTS • ORIGINAL VECTOR DIAGRAMS
VERIFIED PYQs WITH ANSWERS • REVISION + FORMULAS

ABOUT THIS EDITION

Built around the **seven topics printed in the SPPU TE Robotics & Automation 2024 Pattern syllabus**, not around guesses from old papers. Examples are original teaching exercises and are identified as such. This expanded edition separates full concept explanations from the optional numerical workbook. It includes syllabus-mapped older-paper questions with complete model answers and visible source provenance.

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How to use this unit guide

IN SIMPLE WORDS

Understand → draw → derive → solve → write. Read the teal summary at the beginning of a topic, sketch its figure without looking, work the numerical, and then practise writing the navy exam answer with the book closed.

What is official and what is educational support?

- **Official scope:** the seven Unit I topics and course code PCC-303-ROA from SPPU TE Robotics and Automation, 2024 Pattern, effective AY 2026-27, official syllabus printed page 19.
- **Official examination scheme:** End-Semester Examination (ESE) 70 marks; the supplied TE syllabus describes 14 marks per unit. It does *not* guarantee a particular split such as 7+7 for every unit.
- **Practice material:** original worked examples and exam-style questions. Older 2019 Pattern papers are used only for overlapping concepts, not as proof of the current paper format.
- **Formula conventions:** column vectors, right-handed axes, and active rotations; a rotation matrix R acts on coordinates as $p' = Rp$. The distinction between frame changes and rotating vectors is explained explicitly.

COMMON MISTAKE / EXAM TIP

Reading tip for mobile: Open the PDF in a full-page viewer and use *Fit width*; the body font is large and diagrams are vector-drawn for sharp zooming. The contents below link to sections inside this PDF.

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Coverage contract

Official Unit I topic	Where this guide covers it
History and applications	Section 1, industrial and service use-cases
Components and classifications	Section 2, architecture and classifications
Degrees of freedom	Section 3, open and closed-chain mobility
Frames and coordinate systems	Section 4, origins and coordinate changes
Rotation matrices and homogeneous transformation	Sections 5 and 6, derivation and worked problems
Euler angles	Section 7, ZYZ convention and pitfalls
Roll-pitch-yaw representations	Section 8, ZYX convention and numerical comparison

1 History and applications of robotics

IN SIMPLE WORDS

A robot is a reprogrammable machine that senses, computes, and acts to perform physical tasks. Industrial manipulators place or orient tools; mobile robots change their location in the environment.

1.1 From automatic machines to modern robots

1. **Mechanization:** machines supplied force or repeated motion, often following a fixed mechanical sequence.
2. **Programmable automation:** changing a stored program let the same machine perform different operations.
3. **Industrial manipulation:** multi-axis arms combined joints, actuators, end-effectors, controllers and repeatable motion.
4. **Sensor-guided control:** encoders, cameras and force sensors enabled closed-loop correction rather than only pre-recorded motion.
5. **Connected and collaborative systems:** robots can exchange data with industrial controllers and workcells; safety engineering becomes essential when people share space.

Distinguish four ideas: *automation* performs a process with limited human intervention; *robotics* develops machines that sense and act; *kinematics* describes motion without explaining its causes; *dynamics* studies motion caused by forces and torques.

1.2 Why industries use robots

Application	Robot action	Benefit and limitation
Pick-and-place	Locate, grasp, move, release	Repeatable handling; gripper and positioning must match payload
Welding	Move torch along a programmed seam	Consistency; needs fixture accuracy and process safety
Painting	Trace paths with spray tool	Stable coverage; control overspray and hazardous areas
Inspection	Move sensor/camera through viewpoints	Repeatable scanning; accuracy depends on calibration
Medical/service	Move tools or assist people	Fine motion; needs extensive risk controls

WRITE THIS IN THE EXAM

Q. Explain applications of robotics (suggested 6–7-mark answer). Define a robot; state the need for reprogramming, repeatability and sensor feedback; explain pick-and-place, welding, painting, inspection and service robotics with one line each; conclude with benefits (productivity, quality, hazardous-work substitution) and limitations (cost, maintenance, safety, integration).

1.3 What makes a machine a robot?

A washing machine can repeat a sequence but normally has little freedom to alter its physical task. A programmable manipulator changes its motion path and tooling so the same arm can assemble, weld, inspect, or handle parts. The key idea is *programmable physical action*. A robot is not automatically intelligent or autonomous: some industrial robots repeat taught paths without cameras, while others adjust their motion in response to sensors.

Four related terms that examiners often expect you to distinguish:

- **Mechanization** reduces manual effort using a machine. A motorized conveyor is an example.
- **Fixed automation** repeats a predetermined cycle; changing products may need major hardware changes.
- **Programmable automation** changes operations by loading another sequence or program. Industrial robot workcells are an example.
- **Flexible automation** supports faster switching among product families with sensing, coordinated controls and tooling. It is an engineering objective, not a guarantee that every robot is flexible.

IN SIMPLE WORDS

A robot does not need to look like a human. A six-axis arm, a mobile warehouse carrier and a surgical instrument system are all robotic systems because they control physical movement to perform tasks.

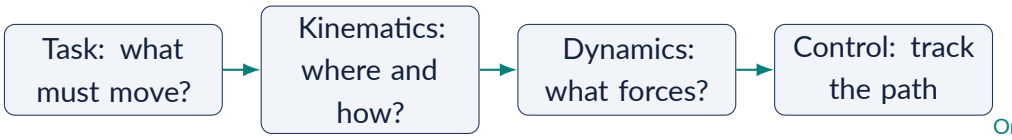
1.4 A practical history of robotics: the engineering changes

1. **Mechanisms and automatic machines.** Cams, gears and linkages produced repeated motions, but changing the motion often required mechanical redesign.
2. **Stored-program industrial robots.** Joint motions could be taught and replayed; a reusable manipulator replaced some dedicated mechanisms.
3. **Closed-loop motion control.** Joint encoders measured actual positions; the controller corrected errors caused by load or disturbance.
4. **Sensing and perception.** Vision and force sensing permitted grasping less precisely located objects and controlling contact with the environment.
5. **Coordinated robotics.** Networked robots, mobile manipulators and collaborative operating modes integrate robot motion with other equipment. Safety functions remain essential.

1.5 Where kinematics sits in the robot-design workflow

A system engineer first defines a task, such as “reach every tray compartment and keep the gripper horizontal”. Next come robot type, required *workspace*, joint limits, tool geometry and cycle time. **Kinematics** answers whether poses can be reached and which joint values or velocities

produce them. **Dynamics** then estimates required torques and whether motors can accelerate the robot. **Control** ensures the planned movement is executed in spite of uncertainties.



Original schematic prepared for Zenith; not to scale.

1.6 Applications explained through one industrial example

Pick-and-place line. A camera identifies a component on a conveyor. The robot converts that location into its own base frame, calculates a reachable pose, moves the gripper to the part, closes it, moves along a safe path and releases the component into a fixture. The following information must be coordinated:

1. *Geometry*: reach, obstacle clearance, part orientation and tool dimensions.
2. *Sensing*: part position, grip confirmation and conveyor timing.
3. *Actuation*: payload capability, speed and repeatability.
4. *Safety*: guarding, emergency-stop circuits and suitable risk controls.

Welding: the torch must follow a path with a suitable orientation, speed and stand-off distance. **Painting:** the spray trajectory must maintain coverage and avoid collisions. **Inspection:** the camera needs proper viewpoints and illumination. **Healthcare and service robotics:** motion must be planned with especially careful operational and safety controls.

COMMON MISTAKE / EXAM TIP

Exam distinction: high *repeatability* means returning consistently near the same taught pose; high *absolute accuracy* means agreeing with the true intended pose. They are related but not identical.

1.7 A complete explanatory answer: benefits and limits

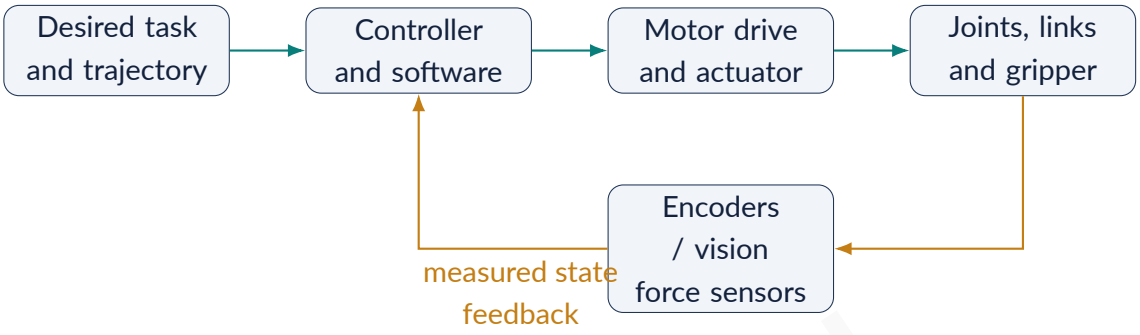
Robotics can increase repeatability, improve process consistency, reduce exposure to harmful environments and enable operations over long shifts. However, a robot is not automatically a good investment. The complete cell may need jigs, integration, sensors, maintenance, safety engineering and trained operators. Task feasibility depends on reach, payload, required accuracy, cycle time, environment and total cost. A strong answer explains both sides rather than stating that robots are always faster or cheaper.

2 Robot components and classifications

IN SIMPLE WORDS

Think of a robot as a loop: a **sensor** measures what happened; the **controller** decides what to do; an **actuator** moves a **link and joint**; the **end-effector** interacts with the task.

2.1 Functional architecture



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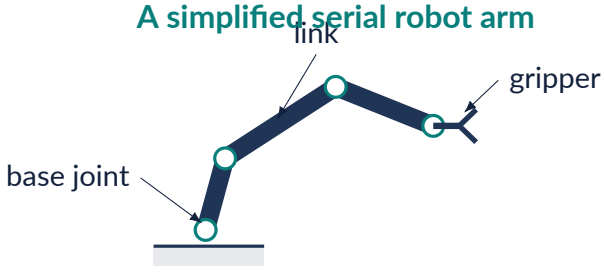
2.2 Physical parts and their roles

- **Base/frame:** mounts the robot; fixes the base reference frame.
- **Links and joints:** rigid members and connections creating the kinematic chain. Revolute joints rotate; prismatic joints translate.
- **Actuators and transmission:** electric motors, pneumatic/hydraulic drives, gear trains, screws or belts supply the required motion.
- **End-effector:** the device at the tool flange, e.g., gripper, welding torch, nozzle or sensor.
- **Sensors:** joint encoders, proximity/limit switches, cameras and force/torque sensors measure state and environment.
- **Controller and power:** hardware and control software execute trajectories, enforce limits and process feedback.

A simplified serial robot arm



Original schematic prepared for Zenith; not to scale.

2.3 Common classifications

Basis	Common categories and identifying feature
Mechanical structure	Cartesian (linear X-Y-Z), cylindrical, spherical/polar, articulated arm, SCARA, parallel manipulator
Motion axes	Revolute (R), prismatic (P), combinations of R and P joints
Drive	Electric, pneumatic, hydraulic (choice depends on force, speed, precision, infrastructure)
Control mode	Point-to-point, continuous-path; open-loop or closed-loop
Operating setting	Industrial, service, mobile, collaborative (requires appropriate risk assessment)

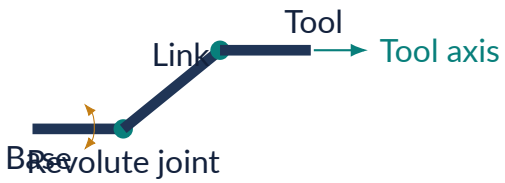
WRITE THIS IN THE EXAM

Q. Draw a robot block diagram and explain components. Draw the sensor–controller–actuator–mechanism loop; label the feedback arrow. Explain base, link, joint, actuator, end-effector, sensor and controller with one sentence per component. Add a pick-and-place example to show how the parts cooperate.

2.4 Mechanical chain: links, joints, base and tool

A **link** is a rigid member, idealized as not deforming during the kinematic calculation. A **joint** constrains how one link moves relative to another. A **revolute joint (R)** allows rotation about one axis and uses angle θ; a **prismatic joint (P)** allows sliding along one axis and uses displacement d. In a serial manipulator, each joint connects successive links from the base to the tool. In a parallel manipulator, several branches support the moving platform.

The **end-effector** is the tool that performs the job: a gripper handles parts, a welding torch joins materials and a probe measures position. Tool size is important: the requested *tool-tip pose* can differ from the flange pose by a fixed tool transform.



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2.5 Actuators and transmissions in simple words

The controller sends a command; an actuator generates force or torque; a transmission carries that motion into the joint. **Electric servo drives** are widely used when accurate controlled motion is required. **Pneumatic drives** can be useful for simple gripper actions where compressed air is available. **Hydraulic drives** can provide high forces but require hydraulic components and maintenance. Gearboxes raise available torque while typically reducing shaft speed. A belt or ball-screw can convert motor rotation into joint translation.

Sensors close the loop. Encoders measure joint position; limit switches identify limits; force/-torque sensors measure interaction; cameras estimate object features; proximity sensors detect presence. Sensor readings can be wrong or noisy, so calibration and filtering matter.

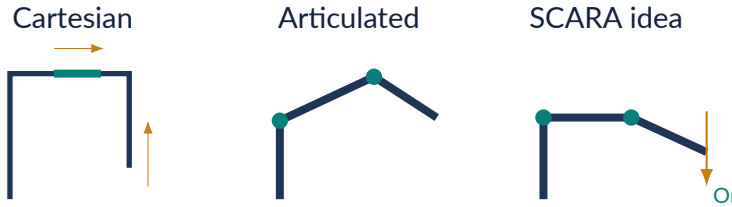
2.6 The feedback loop, step by step

1. The task planner creates desired joint position $q_d(t)$.
2. The encoder provides the measured position $q(t)$.
3. The controller calculates error $e(t) = q_d(t) - q(t)$.
4. The motor driver converts the control signal into electrical drive for the actuator.
5. The joint moves; the sensor reads the new position and the process repeats.

Open-loop systems issue commands without using a measured correction signal. **Closed-loop** systems feed measurement back to reduce error. Closed loop does not mean perfect accuracy: sensors, control tuning, loads and mechanical backlash affect performance.

2.7 Classification by geometry: how to recognize the robot

Configuration	Typical axis pattern	What this geometry makes easy or difficult
Cartesian	P-P-P	Straight motions along fixed axes; structure may occupy a large frame
Cylindrical	R-P-P	Rotational sweep plus two slides; useful around a central work region
Spherical/polar	R-R-P	Two angular directions with radial extension; motion coordinates are not Cartesian
Articulated	R-R-R (often 6R total)	Flexible orientation and approach paths, but kinematics is coupled
SCARA	Commonly R-R-P-R	Fast planar assembly and vertical insertion in suitable workcells
Parallel robot	Multiple linked branches	Potentially stiff and fast in a bounded workspace; special kinematics



Original schematic prepared for Zenith; not to scale.

2.8 Classification by control and work setting

Point-to-point control moves between specified stations, e.g. transfer between fixtures; the intermediate geometric path may not matter to the task. **Continuous-path control** constrains

the path followed by the tool, as in welding or painting. **Industrial versus service** describes where the robot is used. **Collaborative operation** refers to a robot application designed for suitable human-robot interaction with safety measures, not merely a particular arm's shape or marketing label.

WRITE THIS IN THE EXAM

Question: Compare Cartesian, SCARA and articulated robots. A Cartesian manipulator uses orthogonal prismatic motion; this simplifies independent axis positioning. A common SCARA structure has two horizontal rotary joints, a vertical prismatic joint and a tool-axis rotation; it is well suited to many assembly tasks. An articulated manipulator uses several rotary joints, giving flexible tool approaches in a compact reach, but with coupled positioning and more complex collision planning. Classify each with an annotated sketch and one representative application.

ZENITH

3 Degrees of freedom (DoF)

IN SIMPLE WORDS

DoF is the number of independent coordinates needed to specify a robot's configuration. A revolute joint contributes one rotation variable; a prismatic joint contributes one linear-displacement variable, if no constraints remove the independence.

3.1 Body DoF and joint DoF

A free rigid body in **3D space** can translate along three axes (x, y, z) and rotate about three axes (orientation): 6 DoF. In a **plane**, it has 3 DoF: x, y, θ. A serial $3R$ planar arm normally has 3 joint DoF because its three joint angles are independently controlled. Do not confuse *joint DoF* with the dimension of a particular task or workspace.

3.2 Mobility formulas and assumptions

For planar lower-pair linkages the Kutzbach/Grübler count is

$$\boxed{M = 3(n-1) - 2j_1 - j_2}$$

where n is the number of links *including ground*, j_1 is the number of single-DoF lower pairs, and j_2 is the number of higher pairs removing one planar constraint. For a simple spatial linkage with joint freedoms f_i, one commonly writes

$$\boxed{M = 6(n-1-j) + \sum_{i=1}^{j} f_i}$$

where j is the number of joints. Both are **generic constraint counts**; redundant constraints, special geometry, overconstrained mechanisms and dependent joints may require more careful analysis.

SOLVED NUMERICAL

Example 1: mobility of a planar four-bar linkage. There are $n = 4$ links including the fixed frame, $j_1 = 4$ revolute lower pairs and $j_2 = 0$. Therefore

$$M = 3(4-1) - 2(4) - 0 = 9 - 8 = \boxed{1}.$$

Interpretation: one input angle determines the remaining link configuration, except at special configurations that need care.

SOLVED NUMERICAL

Example 2: a 3R planar serial manipulator. Three actuated revolute joints are independent, so $q = [\theta_1, \theta_2, \theta_3]^T$ and $M = \boxed{3}$. Its end-effector pose in the plane is represented by x, y, ϕ, also three quantities; this is not a general rule for all robots.

COMMON MISTAKE / EXAM TIP

Counting mistake: forgetting that n includes the ground link makes a planar four-bar appear to have negative or zero mobility. Another mistake is counting a passive gripper jaw as an independent arm joint when its motion is mechanically coupled.

WRITE THIS IN THE EXAM

Q. Define DoF and derive the mobility of a four-bar. Define DoF; explain independent variables; write and define $M = 3(n-1) - 2j_1 - j_2$; substitute $n = 4, j_1 = 4, j_2 = 0$; show $M = 1$; draw a four-link closed chain with four revolute joints and mark the fixed link. Mention formula limitations.

3.3 Count independent coordinates, not visible parts

Suppose you hold a pen above a table. The tip can be moved in three independent directions; the pen can also be oriented in three independent ways. An unconstrained rigid body in three-dimensional space therefore has six independent pose coordinates. If the pen is constrained to remain flat on the table, its planar pose can be specified by x, y, θ: three degrees of freedom. These numbers describe *possible motion* and do not count motors.

A **robot configuration** is one complete set of independent joint values, collected in $q = (q_1, \ldots, q_n)^T$. A **task-space pose** might have fewer coordinates than the robot's configuration: a seven-joint arm can perform a six-dimensional spatial task with one degree of internal redundancy in a generic nonsingular pose. The relationship depends on constraints and the task definition.

3.4 What each joint contributes

Joint	Allowed relative motion	Configuration coordinate
Revolute (R)	Rotation about a fixed joint axis	Angle θ (rad or degrees)
Prismatic (P)	Translation along a fixed axis	Distance d (m or mm)
Spherical (ideal)	Orientation with three independent rotations	Three orientation parameters, except coordinate singularities
Fixed joint	No relative motion	No independent coordinate

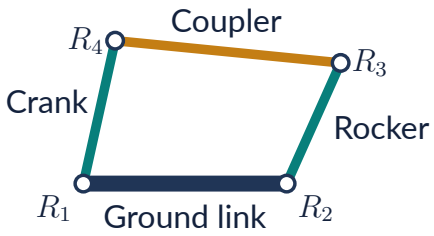
Important: a single geometrical object may have several joint freedoms; conversely, gears, belts and linkages can mechanically couple several visible joint motions to one input.

3.5 Derive the planar mobility count

One unconstrained moving planar link needs three coordinates (x, y, θ). With n links including the fixed ground, there are initially $3(n-1)$ coordinates. Each independent one-DoF lower pair (pin or slider) removes two planar relative freedoms; a typical higher pair removes one. Hence

$$\boxed{M = 3(n-1) - 2j_1 - j_2.}$$

This *count* works for generic planar mechanisms with independent constraints. Special geometry, redundant constraints or inconsistent joint assignments require separate inspection.



Original schematic prepared for Zenith; not to scale.

SOLVED NUMERICAL

Four-bar linkage, with every count explained. The ground, crank, coupler and rocker make $n = 4$ links. Pins at the four corners give $j_1 = 4$, and there are no higher pairs ($j_2 = 0$). Then

$$M = 3(4-1) - 2(4) - 0 = 9 - 8 = \boxed{1}.$$

Give one crank angle; the other angles follow from the closure constraints (away from exceptional configurations).

3.6 How to identify lower and higher pairs

A **lower pair** has surface contact, such as a pin in a bearing or a slider in a guide; for a planar mobility count the standard revolute or prismatic joint is a one-DoF lower pair. A **higher pair** has point or line contact in the ideal model, as with an ideal cam-follower or gear tooth contact. Contact type is not the same as material or size: a small pin remains a lower pair because its kinematic surfaces fit each other.

SOLVED NUMERICAL

Slider-crank mechanism. Count ground, crank, connecting rod and slider ($n = 4$). The joints are the ground-crank pin, crank-rod pin, rod-slider pin and slider-ground prismatic guide ($j_1 = 4$), so $M = 1$. One crank angle controls the piston displacement. Even though the slider moves linearly, the joint is still a lower pair.

SOLVED NUMERICAL

Ideal cam and follower. A simplified planar model uses ground, rotating cam and translating follower: $n = 3$. Cam-ground revolute and follower-ground slider are two lower pairs ($j_1 = 2$), while the cam-follower contacting profiles form one higher pair ($j_2 = 1$). Thus $M = 3(2) - 2(2) - 1 = \boxed{1}$. Assumption: continuous contact is maintained.

3.7 Spatial mobility and its limitations

For generic rigid-link mechanisms moving in 3D, start with six freedoms per moving link. If j joints allow f_i freedoms each, the commonly used constraint count is

$$M = 6(n - 1 - j) + \sum_{i=1}^{j} f_i.$$

For a serial open-chain arm with a grounded base and six one-DoF joints, the count gives $M = 6$: six independently actuated joint coordinates. For closed spatial mechanisms with over-constraints, one must examine the actual independent constraints; plugging in numbers is not enough to establish mobility.

COMMON MISTAKE / EXAM TIP

Exam checklist: write n *including ground*; draw and number every joint; identify joint type; substitute into the correct planar or spatial formula; interpret M in one sentence; mention assumptions when constraints may be dependent.

3.8 Degrees of freedom versus workspace

Workspace is the set of positions (or poses, if orientation is included) the tool can reach under joint limits and geometry. DoF alone does not determine its size. A two-joint planar arm with long links can reach a large area while a six-joint arm with short links may have a much smaller positional reach. **Singular configurations** may remove some instant motion directions, but that does not mean the mechanism permanently loses a physical joint.

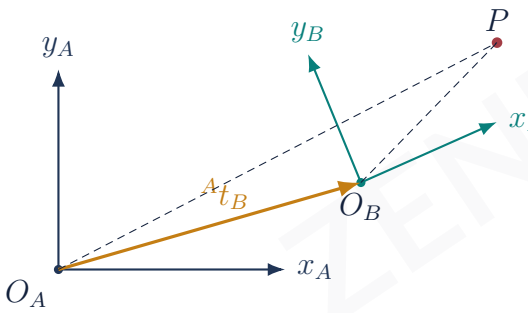
4 Frames and coordinate systems

IN SIMPLE WORDS

A **coordinate frame** is an origin plus mutually perpendicular axes. Frames let us describe the same physical point from different viewpoints without confusing position with orientation.

4.1 Frames on a robot

- $\{W\}$: world frame fixed to the workcell; $\{0\}$: robot base frame; $\{i\}$: link frame; $\{E\}$: end-effector/tool frame.
- ${}^{A}p$ denotes a point's **coordinates expressed in frame A**. Its physical location does not change merely because another frame is used to describe it.
- ${}^{A}R_B$ contains the unit axes of frame B expressed in A. Thus for a pure frame orientation change, ${}^{A}p = {}^{A}R_B\,{}^{B}p$ when the origins coincide.



Original schematic prepared for Zenith; not to scale.

4.2 Active motion versus passive frame change

A numerical matrix may be used in two different sentences. **Active rotation:** a vector physically rotates relative to fixed axes ($p' = Rp$). **Passive change of coordinates:** the vector stays still while its components are re-expressed in rotated axes ($p_B = R^T p_A$ if $R = {}^{A}R_B$ and origins match). Declare which one you are doing before multiplying.

WRITE THIS IN THE EXAM

Q. Explain coordinate frames in robotics. Define origin and orthogonal axes; distinguish world, base, link and tool frames; explain ${}^{A}p$ and ${}^{A}R_B$; draw two shifted/rotated frames; state how frames support robot pose estimation and transform chaining; mention active/-passive convention.

4.3 How a frame is built

A right-handed Cartesian frame consists of an origin O and mutually perpendicular unit axes $\hat{x}, \hat{y}, \hat{z}$. The right-hand rule says $\hat{x} \times \hat{y} = \hat{z}$. A point P can be located by projecting the displace-

ment from the origin onto the axes: ${}^{A}p = [x_A, y_A, z_A]^T$. The superscript tells you *whose rulers and directions* were used to report those numbers.

A robot may use a **world frame** fixed to a workcell, a **base frame** fixed to its mounting, **joint/link frames** attached to each link and a **tool frame** attached to the gripper. Those frames help combine camera measurements, joint geometry and tool offsets without mixing coordinates.

4.4 Position, displacement and direction are different

A **point** has a location relative to an origin; its coordinates change when the origin moves. A **displacement vector** is the difference between two points, so moving the origin alone does not change the vector. A **direction unit vector** tells orientation, with length one, and is not translated. This distinction matters when applying a homogeneous transformation: use homogeneous $w = 1$ for a point, and $w = 0$ for a pure direction.

4.5 Work through a two-frame coordinate conversion

Suppose B has the same orientation as A, but its origin is located at $(3, 1)$ metres in A. Point P lies at $(2, 2)$ metres measured from B. Then the physical point has coordinates ${}^{A}p = (3, 1) + (2, 2) = (5, 3)$ m. If B is rotated 90° counterclockwise relative to A, the B-coordinate $(2, 2)$ becomes $R(90^\circ)(2, 2) = (-2, 2)$ in A directions, then translation gives

$$ {}^{A}p = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 3 \end{bmatrix} \text{ m}}. $$

The physical point did not "rotate by itself" during this change of description; we are converting B coordinates into A coordinates using B's pose in A.

4.6 Frame-notation rules that prevent most errors

1. A rotation ${}^{A}R_B$ takes a vector expressed in B and expresses the same vector in A: ${}^{A}v = {}^{A}R_B \, {}^{B}v$.
2. A transform ${}^{A}T_B$ converts *points* from B coordinates into A coordinates when represented in homogeneous form.
3. For successive frames, the touching letters cancel: ${}^{A}T_C = {}^{A}T_B \, {}^{B}T_C$.
4. To reverse direction, use the inverse: ${}^{B}T_A = ({}^{A}T_B)^{-1}$.
5. Check length units. A matrix may be mathematically correct but physically wrong if one translation is in mm and another in m.

COMMON MISTAKE / EXAM TIP

Active versus passive: $p' = Rp$ can mean physically rotating a vector with respect to fixed axes. In contrast, ${}^{A}p = {}^{A}R_{B}\,{}^{B}p$ can mean expressing an unchanged physical vector in different axes. The numerical matrix may look identical while the sentence it describes is different. State the convention.

4.7 Robot vision case study

A camera detects a part centre at ${}^{C}p = (0.10, 0.02, 0.50)$ m. A calibrated transform ${}^{B}T_{C}$ tells the camera's position and orientation relative to the robot base. The robot should calculate ${}^{B}p = {}^{B}T_{C}[{}^{C}p; 1]$, then choose an approach pose. Without calibration, it would confuse the camera's local axes and origin with the robot's base axes, causing an offset or misoriented grasp. Camera calibration is a practical motivation for learning frames even before forward kinematics.

ZENITH

5 Rotation matrices

IN SIMPLE WORDS

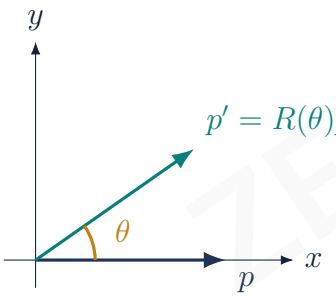
A rotation matrix describes orientation without translation. Its columns are perpendicular unit vectors, so $R^T R = I$ and $\det R = +1$. Inverses are particularly easy: $R^{-1} = R^T$.

5.1 Planar rotation and derivation

In a fixed xy plane, the point (x, y) rotated anticlockwise by angle θ becomes

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{R(\theta)} \begin{bmatrix} x \\ y \end{bmatrix}.$$

This follows from adding angle θ to a point's polar angle and applying the sine/cosine addition rules.



Original schematic prepared for Zenith; not to scale.

5.2 Three fundamental 3D rotations

With right-handed axes and column-vector active rotations:

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix},$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix},$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Important properties: (i) columns and rows are orthonormal; (ii) determinant $+1$; (iii) rotations preserve distances and angles; (iv) composition usually depends on order, $R_x R_z \neq R_z R_x$.

SOLVED NUMERICAL

Example 3: rotating a vector by 30 degrees in the plane. Let $p = [2, 0]^T$. Then

$$R(30^\circ)p = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}}.$$

Check: input length 2; output length $\sqrt{3+1} = 2$, as required for a pure rotation.

SOLVED NUMERICAL

Example 4: rotate $p = [1, 0, 0]^T$ about the z axis by 90 degrees.

$$R_z(90^\circ) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_z(90^\circ)p = \boxed{[0, 1, 0]^T}.$$

COMMON MISTAKE / EXAM TIP

Degrees versus radians: calculator mode must match the angle units. **Order:** the rightmost matrix acts first, e.g. $R_z R_x p$ means rotate about x, then z, when using fixed-frame active rotations.

WRITE THIS IN THE EXAM

Q. Derive a 2D rotation matrix and list its properties. Draw original and rotated coordinates, write sine/cosine angle addition formulas, collect coefficients into the 2x2 matrix, then state orthogonality, inverse-transpose, determinant and length preservation. Illustrate with 90°.

5.3 Build the 2D rotation matrix instead of memorizing it

Take a point with polar radius r and angle ϕ: $x = r\cos\phi$ and $y = r\sin\phi$. Rotate the point counterclockwise by θ around the origin. Its new coordinates are $x' = r\cos(\phi + \theta)$ and $y' = r\sin(\phi + \theta)$. Use the angle-addition identities:

$$x' = r\cos\phi\cos\theta - r\sin\phi\sin\theta = x\cos\theta - y\sin\theta,$$
$$y' = r\sin\phi\cos\theta + r\cos\phi\sin\theta = x\sin\theta + y\cos\theta.$$

Collect coefficients to obtain $\boxed{R(\theta) = \left[\begin{smallmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{smallmatrix}\right]}$. The columns tell us where the original unit x and y directions go after rotation.

5.4 Understand the sign of each entry

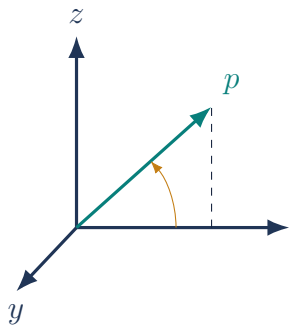
For a small positive θ, $R(\theta)[1, 0]^T$ must point slightly above the positive x axis. It therefore becomes $(\cos\theta, \sin\theta)$ with positive y. This simple mental picture is a reliable way to remember the sign positions. At 90°, $(1, 0)$ becomes $(0, 1)$; at 180° it becomes $(-1, 0)$.

5.5 Properties and why they matter

Orthogonality: the columns of R are perpendicular unit vectors; hence $R^T R = I$. **Inverse:** undoing a rotation is easy because $R^{-1} = R^T$. **Determinant:** a proper rotation has $\det R = +1$; -1 represents a reflection combined with possible rotation, not a proper rotation. **Length preservation:** $\|Rp\|^2 = p^T R^T R p = p^T p$. **Composition:** multiplying $R_2 R_1$ means R_1 acts first on a column vector in the active fixed-axis interpretation.

5.6 Three-dimensional axis rotations explained

For an active right-handed rotation $R_x(\alpha)$, the x component is unchanged while the y, z pair rotate in their plane. $R_y(\beta)$ leaves y unchanged, and $R_z(\gamma)$ leaves z unchanged. If in doubt, apply the matrix to a unit vector to check the result. For example, $R_x(90^\circ)[0,1,0]^T = [0,0,1]^T$.



Right-hand-rule check: point your right thumb along the positive rotation axis. Curled fingers show the positive direction of rotation.

Original schematic prepared for Zenith; not to scale.

5.7 Rotation order is not interchangeable

Start with $p = [1,0,0]^T$. Rotate it first 90° about x, then 90° about z: $R_z(90^\circ)R_x(90^\circ)p = [0,1,0]^T$, because the first x rotation leaves this vector unchanged. Reverse the order: $R_x(90^\circ)R_z(90^\circ)p = [0,0,1]^T$. The different answers prove $R_z R_x \neq R_x R_z$.

SOLVED NUMERICAL

Rotation matrix validation. A candidate $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ has unit, orthogonal columns and determinant $+1$. Therefore it represents a valid proper rotation. It is exactly $R_z(90^\circ)$ and $A^{-1} = A^T = R_z(-90^\circ)$.

5.8 Useful checks for numerical problems

(1) Predict the quadrant before calculating. (2) Check degree/radian mode. (3) Check $R^T R$ and determinant. (4) Confirm output length equals input length for pure rotation. (5) If multiplying rotations, explicitly write which acts first. These five steps find most sign and order mistakes.

6 Homogeneous transformations

IN SIMPLE WORDS

A **homogeneous transformation** combines rotation R and translation t into one 4x4 matrix, making it possible to transform a point and chain several robot frames with matrix multiplication.

6.1 A point transformed from B to A

$$\boxed{{}^{A}T_{B} = \begin{bmatrix} {}^{A}R_{B} & {}^{A}t_{B} \\ 0\,0\,0 & 1 \end{bmatrix}, \qquad \begin{bmatrix} {}^{A}p \\ 1 \end{bmatrix} = {}^{A}T_{B} \begin{bmatrix} {}^{B}p \\ 1 \end{bmatrix}.}$$

R is 3×3, t is 3×1, and the last row is $[0\ \ 0\ \ 0\ \ 1]$. The t column is the location of O_B expressed in A. For multiple frames, chaining works in the order

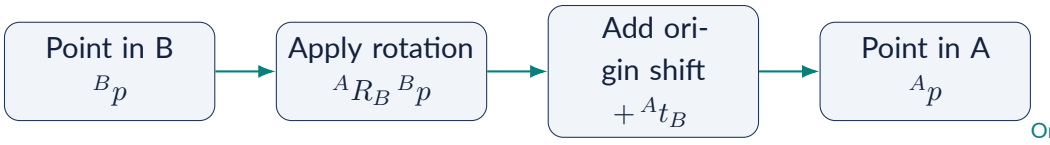
$${}^{A}T_{C} = {}^{A}T_{B}\,{}^{B}T_{C}.$$

6.2 Inverse transform

Because a rotation is orthogonal,

$$\boxed{({}^{A}T_{B})^{-1} = {}^{B}T_{A} = \begin{bmatrix} R^{T} & -R^{T}t \\ 0\,0\,0 & 1 \end{bmatrix}.}$$

It is *not* generally obtained by simply negating the translation; the translation must also be rotated.



Original schematic prepared for Zenith; not to scale.

SOLVED NUMERICAL

Example 5: rotate 90 degrees about z, then translate by $(2, 1, 0)$**.** A point in B is $p_B = (1, 0, 0)^T$. Thus

$$T = \begin{bmatrix} 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}}.$$

Physical interpretation: local x points along world y; the point moves to world position $(2, 2, 0)$.

SOLVED NUMERICAL

Example 6: inverse of the same transformation. With $R^T = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, we compute

$$-R^T t = -\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}.$$

Therefore $T^{-1} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Check by multiplying $T^{-1}(2,2,0,1)^T = (1,0,0,1)^T$.

WRITE THIS IN THE EXAM

Q. Explain homogeneous transformation and solve one point transformation. Define R and t, draw shifted frames, write $T = [R\ t; 0\ 1]$, derive $p_A = Rp_B + t$, show a numeric substitution, and state chain and inverse formulas. Mention how it is used for tool-position calculation in industrial robots.

6.3 Why a 3-by-3 rotation matrix is not enough

A tool tip can move without changing its direction (translation), or rotate without moving its centre (rotation). Robot poses generally combine both. If point ${}^{B}p$ is measured in a tool frame B, and B's origin is displaced by ${}^{A}t_B$ from frame A, then ${}^{A}p = {}^{A}R_B\,{}^{B}p + {}^{A}t_B$. Ordinary matrix multiplication cannot represent the constant addition by a 3-by-3 rotation alone. Introducing a fourth coordinate equal to 1 solves this problem.

$$\begin{bmatrix} {}^{A}p \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} {}^{A}R_B & {}^{A}t_B \\ 0 & 1 \end{bmatrix}}_{{}^{A}T_B} \begin{bmatrix} {}^{B}p \\ 1 \end{bmatrix}.$$

A homogeneous transform is therefore a structured 4-by-4 matrix, not an arbitrary array of sixteen numbers.

6.4 Read the four blocks of a transform

- Upper-left 3×3: R tells how B's axes are oriented in A.
- Upper-right 3×1: t tells where B's origin is located in A.
- Lower-left 1×3: three zeros preserve homogeneous structure.
- Bottom-right: one, making translation possible through multiplication.

6.5 Point versus direction in homogeneous coordinates

A point $[x, y, z, 1]^T$ is rotated *and translated*. A direction $[v_x, v_y, v_z, 0]^T$ is only rotated, because multiplying t by its fourth coordinate gives zero. For example, a camera's viewing direction must not move sideways simply because the camera's position in the workcell changes.

6.6 Composition: three frames, one multiplication

Suppose world W contains robot base B and tool T. If ${}^{W}T_{B}$ is the base pose in world and ${}^{B}T_{T}$ is the tool pose in base, then

$$\boxed{{}^{W}T_{T} = {}^{W}T_{B}\,{}^{B}T_{T}.}$$

The left matrix acts after the right matrix for column vectors. A common robot chain is ${}^{W}T_{E} = {}^{W}T_{0}\,{}^{0}T_{1}\cdots{}^{n-1}T_{n}\,{}^{n}T_{E}$. This is the notation you will use for forward kinematics in Unit II.

SOLVED NUMERICAL

Two translations in sequence. Frame B is translated $+2$ m in A x. Frame C is translated $+3$ m in B y; all axes remain aligned. Then ${}^{A}T_{C}$ has translation $(2, 3, 0)$ m. A point ${}^{C}p = (1, 1, 0)$ m becomes ${}^{A}p = (3, 4, 0)$ m. Check by walking the point through C to B, and then B to A.

6.7 Derive the inverse transform

If $p_A = Rp_B + t$, subtract t: $p_A - t = Rp_B$. Multiply by R^T and use $R^TR = I$:

$$p_B = R^T(p_A - t) = R^Tp_A - R^Tt.$$

Thus $T^{-1} = \left[\begin{smallmatrix} R^T & -R^Tt \\ 0 & 1 \end{smallmatrix}\right]$. The translation is $-R^Tt$, not usually $-t$. You can verify the inverse by showing $TT^{-1} = I_4$.

6.8 Robot-tool calibration case

A gripper flange may be 8 cm behind the actual fingertip along the tool's local z axis. Represent the flange-to-tip relationship as ${}^{F}T_{P} = [I, (0, 0, .08)^T; 0, 1]$ m. If the flange points sideways relative to the base, that 8 cm offset rotates into a different base direction: ${}^{B}T_{P} = {}^{B}T_{F}\,{}^{F}T_{P}$. Adding $(0, 0, .08)$ directly to a base-frame translation would be wrong unless the frames happen to be aligned.

6.9 Exam-style step-by-step method

1. Sketch the two frame axes and mark the origin displacement.
2. Form R from the orientation, checking the right-hand rule.
3. Write t in the *destination* frame's coordinates and consistent units.
4. Assemble $T = [R\ t;\ 0\ 1]$; multiply a homogeneous point.
5. If reversing the mapping, compute R^T and $-R^Tt$ separately.

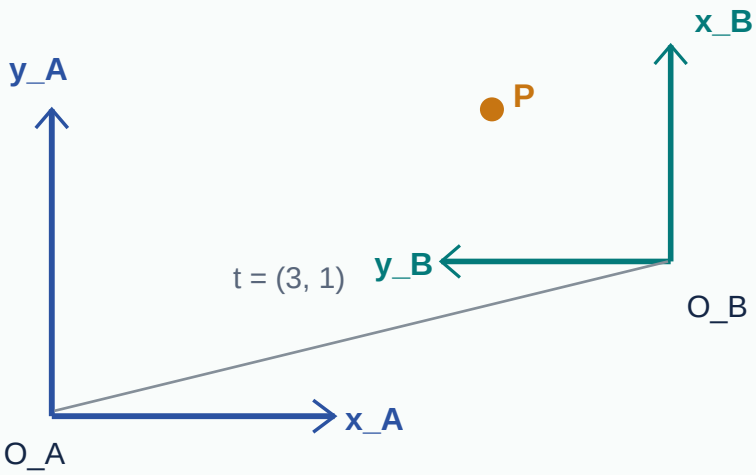
6. State the final point and its units; check an easy special case if possible.

COMMON MISTAKE / EXAM TIP

Do not add a tool offset in a mismatched coordinate frame. The translation column belongs to the destination frame. A local tool offset must be rotated before it is added in the base frame.

REVISION WORKSPACE | HOMOGENEOUS TRANSFORMS

Extra Zenith teaching example (not a previous-year question). Practise the frame equation from Section 6 before moving to Euler angles.



Sketch is schematic; the numerical rotation and translation below define the model.

WORKED EXAMPLE | CHANGE COORDINATES

Frame B is rotated +90 degrees relative to A. Its origin is t=(3,1) m in A. A point has coordinates p_B=(2,2) m. Find p_A.

Step 1: R(90 deg) p_B = (-2, 2) m.
Step 2: Add origin shift: p_A = (-2, 2) + (3, 1).
Answer: p_A = (1, 3) m.

Exam check: Transform the local origin p_B=(0,0). Its A coordinates must equal t=(3,1). Translation changes points but not pure directions.

7 Euler angles: orientation through three rotations

IN SIMPLE WORDS

Euler angles represent a 3D orientation with **three sequential rotations**. They are useful but *not unique*: you must state the axes and rotation order. In this guide, the Euler example uses the **ZYZ** sequence.

7.1 ZYZ Euler convention

For fixed-frame active matrices, a ZYZ orientation is represented as

$$\boxed{R_{ZYZ}(\alpha, \beta, \gamma) = R_z(\alpha)R_y(\beta)R_z(\gamma).}$$

Reading right-to-left for the matrix acting on a vector: apply a z rotation by γ, then a y rotation by β, then a z rotation by α. Intrinsic moving-axis descriptions of Euler rotations are often stated in the opposite chronological narrative, so identify the convention when presenting a solution.

7.2 Why three rotations?

A rigid body's orientation has three independent local parameters away from singular representations. Two angular variables are insufficient for an arbitrary orientation; three sequential rotations can represent it, although the mapping from angles to orientation can become non-unique at special configurations.

SOLVED NUMERICAL

Example 7: special ZYZ Euler case. Take $\alpha = 30°$, $\beta = 0°$, $\gamma = 45°$. Since $R_y(0) = I$,

$$R = R_z(30°)R_z(45°) = \boxed{R_z(75°)}.$$

Interpretation: the first and third z rotations become indistinguishable when the middle angle is zero. This is a representation singularity (often called gimbal lock), not the robot necessarily losing a mechanical joint.

COMMON MISTAKE / EXAM TIP

Textbooks use multiple Euler sequences (e.g. ZYZ and ZXZ) and active/passive conventions. **Do not memorise one product without its axis labels.** An answer that says only "Euler angles = roll, pitch, yaw" is not generally correct.

WRITE THIS IN THE EXAM

Q. Explain Euler-angle representation of orientation. Define three sequential rotations; declare the ZYZ convention, draw axes, write $R = R_z(\alpha)R_y(\beta)R_z(\gamma)$; explain order and non-uniqueness; use $\beta = 0$ to illustrate gimbal lock; state industrial robot orientation as a use-case.

7.3 Why three angles can describe an orientation

A spatial orientation has three independent local freedoms, so it is natural to describe it by three successive rotations. But three numbers are not enough to specify a convention: you must also state the axes, the order, and whether axes are fixed or moving. The term *Euler angles* is used for sequences such as ZYZ or ZXZ; the common ZYX yaw-pitch-roll convention is usually called a Tait–Bryan sequence, although some software loosely groups all three-angle descriptions together.

7.4 Intrinsic ZYZ: do the rightmost rotation first

In this guide, intrinsic ZYZ Euler angles use a rotation by α about the starting z axis, then β about the new y axis, then γ about the newest z axis. With the chosen active column-vector convention the combined orientation matrix is

$$\boxed{R_{ZYZ} = R_z(\alpha)R_y(\beta)R_z(\gamma).}$$

When you use the product as an *active fixed-frame multiplication on a vector*, the rightmost factor acts first. Both descriptions are consistent only when you keep track of the moving-axis versus fixed-axis interpretation.

7.5 How to obtain the matrix entries

Let $c_a = \cos\alpha$, $s_a = \sin\alpha$, and likewise c_b, s_b, c_g, s_g. Multiplying the three elementary matrices gives

$$R_{ZYZ} = \begin{bmatrix} c_a c_b c_g - s_a s_g & -c_a c_b s_g - s_a c_g & c_a s_b \\ s_a c_b c_g + c_a s_g & -s_a c_b s_g + c_a c_g & s_a s_b \\ -s_b c_g & s_b s_g & c_b \end{bmatrix}.$$

Use this expanded form when a question supplies a complete orientation matrix. For routine numericals, the product form is easier to understand and produces fewer transcription errors.

SOLVED NUMERICAL

ZYZ numerical with a special angle. Let $\alpha = 0°$, $\beta = 90°$, $\gamma = 0°$. Then $R_{ZYZ} = R_y(90°) = \left[\begin{smallmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{smallmatrix}\right]$. Applying it to $[1, 0, 0]^T$ gives $[0, 0, -1]^T$. The result makes physical sense: positive rotation about y sends $+x$ toward $-z$.

7.6 Gimbal lock is a coordinate-description problem

At $\beta = 0$ in ZYZ, the middle rotation is identity and the first and third z rotations combine: $R = R_z(\alpha+\gamma)$. You can measure their sum but cannot uniquely determine both angles. A similar degeneracy occurs at $\beta = \pi$. This is a *singularity of the angle parameterization*: the physical robot need not be jammed, and the same orientation still has a valid rotation matrix. A mathematically robust system may use matrices or quaternions internally rather than relying solely on these angles.

COMMON MISTAKE / EXAM TIP

Do not say that every Euler sequence is ZYX. Explicitly name the convention. ZYZ and ZYX represent the same possible physical orientations in different coordinates, but their numbers and singular conditions differ.

7.7 Exam answer pattern

Define orientation parameterization, list the ZYZ rotation sequence, draw the successive frame axes, write $R_z(\alpha)R_y(\beta)R_z(\gamma)$, explain matrix multiplication order, give one numerical and identify the nonuniqueness when $\beta = 0$ or π. End with an application such as describing tool orientation during a manipulator's motion.

ZENITH

8 Roll-pitch-yaw (RPY) representations

IN SIMPLE WORDS

Roll, pitch and yaw are rotations conventionally associated with the x, y, and z axes. A frequently used aerospace/robotics convention is **ZYX**, written as the matrix product $R_z(\psi)R_y(\theta)R_x(\phi)$.

8.1 Define one unambiguous ZYX convention

Using fixed-frame active rotations and column vectors, define roll ϕ, pitch θ and yaw ψ through

$$\boxed{R_{\text{RPY}} = R_z(\psi)R_y(\theta)R_x(\phi).}$$

The rightmost factor acts first on a vector. One may instead describe equivalent intrinsic (body-axis) rotations, so naming the axes and product order is essential. ZYX RPY and ZYZ Euler are different parameterizations: do not substitute one for the other.

8.2 Physical meaning

- **Roll** ϕ: tilt about the x axis.
- **Pitch** θ: tilt about the y axis.
- **Yaw** ψ: heading turn about the z axis.

SOLVED NUMERICAL

Example 8: yaw-only tool orientation. Roll $\phi = 0$, pitch $\theta = 0$, yaw $\psi = 90°$. Then

$$R = R_z(90°) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The tool-frame local x direction $[1, 0, 0]^T$ points along world $+y$: $R[1, 0, 0]^T = [0, 1, 0]^T$.

SOLVED NUMERICAL

Example 9: combined rotation. If roll $\phi = 90°$, pitch $\theta = 0$, yaw $\psi = 90°$,

$$R = R_z(90°)R_x(90°) = \boxed{\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}.$$

Multiply $R^T R$ to confirm I; verify $\det R = 1$. Because the operations do not commute, reversing the order generally produces another orientation.

8.3 RPY singular configurations

At $\theta = \pm 90^\circ$ in the ZYX convention, roll and yaw cannot be determined independently from orientation alone. This is *gimbal lock*; it is a coordinate-parameterization issue. Quaternions or rotation matrices can avoid this parameterization problem, but they are extensions beyond the explicit list of Unit I topics.

WRITE THIS IN THE EXAM

Q. Describe RPY orientation representation. Define roll, pitch and yaw with axes; state ZYX product $R_z(\psi)R_y(\theta)R_x(\phi)$; sketch axes; give yaw-only example; distinguish it from a ZYZ Euler sequence and mention gimbal lock at $\theta = \pm 90^\circ$.

Extended worked tutorial: build the ideas yourself

Tutorial A: read the anatomy of an industrial robot

Consider a six-axis pick-and-place robot. Its base locates it in the cell; its shoulder and elbow contribute much of the reach; wrist joints orient the tool; a gripper closes on an object; encoders measure joint angles; and a controller compares actual angles with demanded angles. If a box is misplaced, the cause might be a fixture shift, faulty camera calibration, wrong tool frame, slipping gripper, or a trajectory error. This teaches an important principle: **kinematic accuracy is not identical to task success**. The model may place the tool at the commanded pose while the gripper still drops the payload.

WRITE THIS IN THE EXAM

Model answer: Why are sensors necessary in an industrial robot? A sensor converts physical conditions to measured signals. Joint encoders estimate motor/link position, limit switches protect travel boundaries, cameras estimate workpiece pose, and force/torque sensors help detect contact. Feedback makes it possible for the controller to compare a reference with the measured state, correct errors, and respond to disturbances. An open-loop system has no such measurement-based correction.

Tutorial B: deciding whether a motion is truly independent

Suppose a gripper has two opposing jaws connected by gears so they open symmetrically. Though two visible pieces move, one motor variable determines both jaw positions. Thus the jaws add only *one independent gripper coordinate*, not two. By contrast, a robot with two independent prismatic slides along orthogonal axes needs two variables, $q_1 = x$ and $q_2 = y$. Ask: *Can one motion change while all remaining coordinates stay fixed?* If yes, it may be an independent DoF.

SOLVED NUMERICAL

Example 10: mobility of a planar slider-crank. Ground, crank, connecting rod and slider give $n = 4$ links. There are three revolute pairs and one prismatic pair: $j_1 = 4$. There are no higher pairs: $j_2 = 0$. Thus

$$M = 3(4 - 1) - 2(4) = \boxed{1}.$$

Even though the slider moves linearly and the crank rotates, both motions are coupled by the linkage, so one input is sufficient.

Tutorial C: the meaning of the columns of a rotation matrix

Let the local unit directions be $\hat{x}_B = [1, 0, 0]^T$, $\hat{y}_B = [0, 1, 0]^T$, $\hat{z}_B = [0, 0, 1]^T$. Expressing them in world frame A gives three vectors $\hat{x}_B^A, \hat{y}_B^A, \hat{z}_B^A$. Place them as columns:

$${}^A R_B = \begin{bmatrix} | & | & | \\ \hat{x}_B^A & \hat{y}_B^A & \hat{z}_B^A \\ | & | & | \end{bmatrix}.$$

This geometric meaning helps you *build* a matrix rather than memorise it. For a positive 90° rotation about z, local x points along world +y, local y points along world -x, and local z still points along world +z. Thus the columns are $[0, 1, 0]^T$, $[-1, 0, 0]^T$, $[0, 0, 1]^T$, producing the standard $R_z(90^\circ)$.

SOLVED NUMERICAL

Example 11: test whether a proposed orientation matrix is valid.

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Its columns each have unit length and their dot products are zero. Hence $A^T A = I$. Expanding the determinant gives $\det A = 1$, so A is a proper rotation matrix. If the determinant were -1 it would represent a reflection or improper orthogonal transform, not a pure rotation.

Tutorial D: where the translation column comes from

Imagine an end-effector frame B whose origin is 20 cm in world x and 10 cm in world y. Even if B's axes are parallel to world axes, its origin has shifted: ${}^A t_B = (.20, .10, 0)^T$ metres. A point at B's origin has coordinates ${}^B p = (0, 0, 0)^T$; applying ${}^A p = R\,{}^B p + t$ gives ${}^A p = t$, exactly as expected. A point 5 cm forward along local B x has ${}^B p = (.05, 0, 0)^T$ and world position $(.25, .10, 0)^T$ if the axes are parallel. This simple check catches the common mistake of confusing point coordinates with the position of the frame origin.

SOLVED NUMERICAL

Example 12: build a transform from pose data. Frame B is aligned with A but its origin is at $(.2, .1, .3)$ m. Therefore

$$ {}^{A}T_{B} = \begin{bmatrix} 1 & 0 & 0 & .2 \\ 0 & 1 & 0 & .1 \\ 0 & 0 & 1 & .3 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

For ${}^{B}p = (.05, .02, 0)^{T}$ m, the world point is $\boxed{(.25, .12, .30)^{T} \text{ m}}$. If you obtain the original B coordinates, you forgot to apply translation.

Tutorial E: extracting orientation information safely

Before interpreting reported angles, record the sequence. In a ZYX yaw-pitch-roll system, $R = R_z(\psi)R_y(\theta)R_x(\phi)$. A yaw-only rotation changes heading while keeping vertical direction fixed. A pitch-only rotation can tilt a tool's forward axis toward vertical. A roll-only rotation spins about the local forward direction in the corresponding physical interpretation. The same physical orientation can be described by a rotation matrix and more than one set of angle values. The matrix is the safer common language between conventions.

SOLVED NUMERICAL

Example 13: an orientation consistency check. For zero RPY angles, the ZYX formula gives $R_z(0)R_y(0)R_x(0) = I$. For yaw 180° and zero pitch/roll,

$$ R = \text{diag}(-1, -1, 1). $$

It maps world $+x$ to $-x$, $+y$ to $-y$, and leaves $+z$ unchanged, matching a half-turn about the vertical axis.

Tutorial F: how to structure a full descriptive exam answer

For a question on **homogeneous transformations**, a clear answer can follow this six-part sequence:

1. **Definition:** a 4x4 representation of relative orientation and position of rigid frames.
2. **Neat figure:** draw frames A and B, their origins and a point P.
3. **Matrix:** show block rotation R, translation t, zeros and final 1.
4. **Physical meaning:** describe axes mapping and origin shift.
5. **One calculation:** transform a point with explicit units and a final boxed answer.
6. **Application:** tool-frame position in a workcell or combining link transforms.

This structure is a study suggestion, not a university-mandated marking rubric. It makes reasoning visible to an examiner and reduces the chance of writing an unexplained formula.

ONE-NIGHT REVISION

Self-check before moving to Unit II: If someone hands you a diagram of two coordinate frames, can you identify the translation vector and each column of the rotation matrix? If someone gives you a 4x4 matrix, can you tell whether its 3x3 part is a proper rotation? If someone says "Euler angles", do you ask for the axis sequence? These questions are more revealing than memorising formulas alone.

8.4 Yaw, pitch and roll: attach each word to an axis

For the ZYX convention used here, **yaw** ψ is a rotation about z, **pitch** θ about the subsequent y, and **roll** ϕ about the subsequent x axes. With the standard column-vector representation:

$$\boxed{R_{ZYX} = R_z(\psi)R_y(\theta)R_x(\phi).}$$

The words yaw, pitch and roll are common in aircraft and mobile robot descriptions, but the precise frame and sign conventions may differ across applications. Always copy the convention stated by the problem or declare the one used.

8.5 Derive and read the full ZYX matrix

Define $c_\psi = \cos\psi$, $s_\psi = \sin\psi$, etc. Multiplying the three basic rotation matrices yields

$$R_{ZYX} = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix}.$$

The first column is the local x axis expressed in the world frame; the third column is the local z axis. For zero angles, $R = I$; for yaw-only motion, the expression reduces to $R_z(\psi)$.

8.6 How to calculate RPY angles from a matrix

For a matrix $R = (r_{ij})$ with $|\cos\theta| > 0$ and the convention above, a common principal solution is

$$\theta = \text{atan2}(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}), \quad \psi = \text{atan2}(r_{21}, r_{11}), \quad \phi = \text{atan2}(r_{32}, r_{33}).$$

The $\text{atan2}(y, x)$ function handles quadrants more reliably than $\arctan(y/x)$. At $\theta = \pm 90°$, $\cos\theta = 0$ and these simple formulas cannot independently identify yaw and roll. Software must handle this singular case using a declared convention and continuity rules.

SOLVED NUMERICAL

RPY extraction. Suppose $R = \left[\begin{smallmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{smallmatrix}\right]$. The third row gives $\theta = \text{atan2}(0, 1) = 0$, the first column gives $\psi = \text{atan2}(1, 0) = 90°$, and the last row gives $\phi = \text{atan2}(0, 1) = 0$. So one valid principal RPY triple is $(\phi, \theta, \psi) = (0, 0, 90°)$.

8.7 Understand RPY through a robotic gripper

Imagine a gripper pointing forward along its local x axis. **Yaw** changes the heading direction in the workcell, **pitch** tilts its forward direction upward or downward, and **roll** spins around its own longitudinal axis. To pick a bolt from a shelf, the gripper may require a particular pitch and roll rather than only its tip position. This is why robotic manipulation uses *pose* (position plus orientation), not just an (x, y, z) target.

8.8 Comparison of ZYZ and ZYX in one table

Feature	ZYZ Euler	ZYX yaw-pitch-roll
Axes	Z, Y, Z	Z, Y, X
Product	$R_z(\alpha)R_y(\beta)R_z(\gamma)$	$R_z(\psi)R_y(\theta)R_x(\phi)$
Typical labels	α, β, γ	yaw ψ, pitch θ, roll ϕ
Angle singularity	$\beta = 0, \pi$	$\theta = \pm\pi/2$
Same physical object?	Yes, one possible description of its orientation	Yes, a different description of the same orientation

COMMON MISTAKE / EXAM TIP

Most common RPY error: writing the order as $R_xR_yR_z$ because the spoken names are "roll, pitch, yaw". With our ZYX matrix convention, the correct product is $R_z(\psi)R_y(\theta)R_x(\phi)$.

9 Essential comparisons and common mistakes

Concept	What it means	Remember
Kinematics vs dynamics	Geometry of motion vs forces/-torques causing motion	Unit I is about configuration and pose
Revolute vs prismatic	One angular variable vs one linear variable	Both often contribute one DoF
Position vs orientation	Location of origin vs axes direction	Use t for position, R for orientation
Rotation vs transform	3x3 orientation vs 4x4 pose	Homogeneous T includes translation
Euler ZYZ vs RPY ZYX	Different chosen angle sequences	Always specify convention
Joint DoF vs task dimension	Independent joint coordinates vs task coordinates	They need not match

COMMON MISTAKE / EXAM TIP

Exam checklist: label all coordinate frames; mark positive rotation with the right-hand rule; draw a 4x4 homogeneous matrix with the bottom row; specify the order for Euler/RPY; verify $R^T R = I$; write units and a final numerical answer clearly.

10 Previous-year questions: verified overlap and complete answers

COMMON MISTAKE / EXAM TIP

Source and pattern warning. The older original papers checked for this section explicitly say **2019 Pattern**. They are not 2024 Pattern PYQs and their mark allocations are *not* predictions for the TE 2024 Pattern Semester V examination. The questions below are selected because their concepts overlap the seven official Unit I topics. Where wording is summarized, that is clearly marked; source IDs and PDF pages let you inspect the originals.

PYQ A: hardware architecture of a robotic system (8 marks)

Original source: SPPU RKD, April 2024, paper **6262-175**, **Q7(b)**, physical PDF p. 3, 2019 Pattern. *Question gist:* Identify the key components and subsystems in a robotic system's hardware architecture and explain their cooperation. View source PDF.

WRITE THIS IN THE EXAM

Model answer. A robotic system is a controlled mechanical assembly capable of moving an end-effector to perform a task. Its main physical subsystems are: (1) **mechanical structure** including base, links and joints; (2) **actuation** such as servo motors and associated transmission; (3) **sensing** such as joint encoders, limit switches, cameras and force sensors; (4) **control hardware** that reads sensors and computes commands; (5) **power and drives** that deliver controlled electrical, hydraulic or pneumatic energy; (6) **end-effector** that contacts the workpiece; and (7) **human interface and safety system** such as teach pendant, limits and emergency stop.

Operation: The programmer defines a target. The controller converts it into joint commands. Motor drives actuate the joints, causing the mechanical arm and tool to move. Sensors report actual joint positions and process conditions. In closed loop the controller compares measured and target positions and corrects error. Safety functions monitor hazardous conditions and command appropriate safe behaviour.

Industrial example: In pick-and-place, vision may detect a component, the robot calculates tool pose, the servo motors move the arm and a gripper holds the component. A clear block diagram shows *task* → *controller* → *drive* → *manipulator* → *tool* with a feedback arrow from sensors to the controller.

Conclusion: Robot performance depends on coordinated mechanical, sensing, power, control, tooling and safety design, not on the robot arm alone.

PYQ B: pick-and-place and applications (8 marks)

Original source: SPPU RKD, November 2025, paper **6583-177**, **Q4(b)**, physical PDF p. 1, 2019 Pattern. *Question gist:* Explain a pick-and-place operation and its applications. View source PDF.

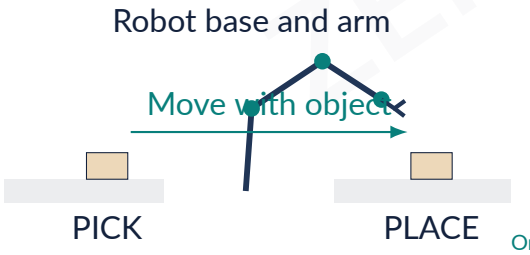
WRITE THIS IN THE EXAM

Model answer. *Pick-and-place* is a repetitive robotic operation in which a robot locates an object, positions a tool to grip it, transfers the object to a new location and releases it. It is common in packaging, bin sorting, machine loading, electronics assembly and warehouse handling.

Working cycle: (1) Detect or receive the object's pickup pose. (2) Move the arm to a safe approach pose above the object. (3) Lower or approach the gripper while maintaining required orientation. (4) Close fingers or apply vacuum, then verify grip if a sensor is available. (5) Lift and follow a collision-free transfer path. (6) Approach the destination fixture. (7) Release the object and retreat to a safe pose.

Important components: manipulator links and joints determine reach; the gripper must match size, weight and surface; position sensors and encoders improve repeatability; vision may correct changing part positions; the controller coordinates timing and motion. **Advantages** include consistent repetitive handling and reduced exposure to hazardous tasks. **Constraints** include payload limit, gripper design, speed, reach, safety and part localization.

Neat diagram to draw: two workstations labeled PICK and PLACE, a robot arm between them, arrows for approach, transfer and retreat, and a label on the gripper.



Original schematic prepared for Zenith; not to scale.

PYQ C: the PUMA robot and joint freedoms (9 marks)

Original source: SPPU RKD, November 2025, paper **6583-177**, **Q8(a)**, physical PDF p. 2, 2019 Pattern. *Question gist:* Describe the PUMA structure, DoF and joint types. View source PDF.

WRITE THIS IN THE EXAM

Model answer. PUMA stands for *Programmable Universal Machine for Assembly*. A conventional PUMA-style industrial manipulator is a serial **six-revolute-joint (6R)** articulated robot. It is commonly understood as an arm with three joints mainly locating the wrist centre and a three-axis wrist mainly orienting the tool. Its six independent joint angles normally provide six joint DoF (subject to limits and particular configurations).
Main groups: the **waist** rotates the arm about the base, the **shoulder and elbow** help position the wrist, and the **wrist joints** change tool orientation. The end-effector is attached to the last link. All six ideal arm joints are revolute; the gripper may have its own separate opening and closing mechanism which should not casually be counted as an extra arm positioning joint.
Applications: machine tending, assembly and manipulation where orientation of a tool matters. **Limitations:** the finite workspace, joint limits, payload limits and singular configurations can restrict available motions. Draw a six-joint articulated arm, label joint axes J_1 through J_6, and distinguish the gripper from the six arm joints.

PYQ D: tool configuration for an articulated robot (9 marks)

Original source: SPPU RKD, April 2024, paper **6262-175**, **Q1(a)**, physical PDF p. 1, 2019 Pattern.
Question gist: Explain the tool configuration of a five-axis articulated robot. View source PDF.

WRITE THIS IN THE EXAM

Model answer. The *tool configuration* is the tool's position and orientation in a chosen reference frame. Position is specified by x, y, z and orientation can be represented by a rotation matrix or a declared three-angle representation. A **five-axis articulated robot** has five independently controlled motion axes in the usual interpretation. Its geometry may allow many tool positions but impose limits on independently achievable orientations compared with a general nonsingular six-axis manipulator.
To describe a tool pose, establish a base frame $\{B\}$, attach a tool frame $\{T\}$ at the tip, then determine ${}^{B}T_{T} = \left[\begin{smallmatrix} R & t \\ 0 & 1 \end{smallmatrix}\right]$. The translation $t = (x, y, z)^{T}$ locates the tool origin in the base; the columns of R express the tool axis directions. The transforms of intermediate links determine ${}^{B}T_{T}$ in forward kinematics (Unit II). Five controlled axes do not imply exactly "five Cartesian coordinates"; reachable task poses depend on joint geometry and constraints.
Applications: an appropriately constrained welding, palletizing or handling task where a tool need not achieve every possible spatial orientation. Show a labeled five-axis arm and a tool frame. Explain position, orientation and what each commanded joint contributes.

10.1 Why some other old questions were excluded

Questions about genetic algorithms, advanced inverse kinematics, robot balancing, gripper magnetic-force sizing and dynamic equations appear in the inspected 2019 Pattern papers but are *not* part of the seven official RKD Unit I headings. Some are relevant to later units, electives or old curricular scopes. Zenith does not force them into this unit merely to increase page count.

For every future PYQ addition, the workflow must retain paper number, pattern, Q-number, original PDF page, verified topic mapping and source availability.

11 Expanded question bank with answer checkpoints

How to use: these are Zenith-created practice questions, *not represented as actual SPPU past-paper questions*. Attempt each question with the guide closed. A good descriptive answer has a definition, explanation, neat labeled diagram or matrix, application and conclusion. Calculations should show assumptions, substitution, units and checks.

Part A: understanding and diagrams

QB1. Explain the difference between mechanization, fixed automation, programmable automation and a robot. **Checkpoint:** ability to change the physical task with software, plus actuator/sensor/control roles.

QB2. Give five real robotic applications and one technical requirement for each. **Checkpoint:** pick-and-place/gripper, welding/tool path, painting/coverage, inspection/sensing, healthcare/safety.

QB3. Draw a robot functional block diagram with feedback. **Checkpoint:** task, controller, drives, joints/link, tool, sensor feedback, power and safety.

QB4. Explain revolute and prismatic joints with sketches and examples. **Checkpoint:** one joint variable, θ versus d, their directions and units.

QB5. Compare Cartesian, cylindrical, SCARA and articulated robots. **Checkpoint:** axis structure, advantages, applications and limitations.

QB6. Define DoF, joint space, task space and workspace. **Checkpoint:** independent joint variables need not equal task coordinate count.

QB7. Why does a rigid body need six pose freedoms in 3D but three in a plane? **Checkpoint:** translations and rotations and associated constraints.

QB8. Explain mobility of a four-bar, including links and joints drawn. **Checkpoint:** $n = 4$, $j_1 = 4$, $j_2 = 0$, $M = 1$.

QB9. Differentiate a lower pair from a higher pair. **Checkpoint:** ideal contact geometry and representative joints.

QB10. Define world, base, link and tool coordinate frames. **Checkpoint:** origin, axes, units and role in calibration.

QB11. Explain active rotation and passive coordinate conversion. **Checkpoint:** whether physical vector or coordinate basis changes.

QB12. Explain the physical meaning of all sixteen positions in a homogeneous transform. **Checkpoint:** 3×3 R, 3×1 t, zeros, last 1.

QB13. Compare ZYZ Euler angles and ZYX RPY. **Checkpoint:** axes, product order, angle names and singular cases.

Part B: derivations and solved-answer checks

QB14. Planar rotation. Derive $R(\theta)$ from angle-addition formula. Rotate $p = (4, 0)$ by $30°$. **Answer:** $(2\sqrt{3}, 2)$; length 4 preserved.

QB15. Mobility. Find mobility of a slider-crank with four links, four lower pairs and no higher pair. **Answer:** $M = 3(3) - 2(4) = 1$.

QB16. Cam-follower. Use three links, two lower pairs and one higher pair. **Answer:** $M = 3(2) - 4 - 1 = 1$ under maintained contact.

QB17. Rotation. $R_z(90°)$ acts on $(0, 3, 0)$. **Answer:** $(-3, 0, 0)$. Check vector length.

QB18. Rotation order. Compare $R_z(90°)R_x(90°)e_x$ with $R_x(90°)R_z(90°)e_x$. **Answers:** e_y versus e_z.

QB19. Frame conversion. B is rotated $90°$ and translated $(3, 1)$ relative to A. Find A coordinates of ${}^{B}p = (2, 2)$. **Answer:** ${}^{A}p = (1, 3)$.

QB20. Transform. Rotate $(1, 2, 0)$ by $R_z(90°)$ and then translate $(4, -1, 2)$. **Answer:** $(2, 0, 2)$.

QB21. Inverse. For $R = R_z(90°)$ and $t = (2, 1, 0)$, calculate $-R^T t$. **Answer:** $(-1, 2, 0)$; show the full inverse.

QB22. Chaining. A-to-B translation is $(2, 0, 0)$ and B-to-C is $(0, 3, 0)$, with aligned axes. **Answer:** A-to-C translation $(2, 3, 0)$.

QB23. ZYZ. Evaluate $R_z(0)R_y(90°)R_z(0)$ and apply it to e_x. **Answer:** $(0, 0, -1)$.

QB24. RPY. Extract yaw, pitch and roll for $R_z(90°)$. **Answer:** yaw $90°$, pitch 0, roll 0 in the chosen convention.

QB25. Conceptual diagnosis. A student says a vector becomes longer after a pure rotation. How do you reject the result? **Checkpoint:** prove $\|Rp\| = \|p\|$ using $R^T R = I$.

Extended answer-writing tasks

E1. Write an 8-mark answer for robot hardware architecture using at least seven correctly labeled parts; compare your answer with PYQ A.

E2. Write an 8-mark pick-and-place explanation including the sequence, applications, benefits and limitations; compare with PYQ B.

E3. Draw and explain PUMA's six-revolute-joint idea; distinguish arm joint DoF from gripper jaw motion; compare with PYQ C.

E4. Explain tool position and orientation using two coordinate frames and ${}^{B}T_{T}$; compare with PYQ D.

ONE-NIGHT REVISION

Self-assessment: mark each of the seven official topics as (1) understand it, (2) draw its diagram, (3) solve a typical problem, and (4) write an answer without looking. Revisit any topic missing one of those four capabilities.

12 Last-night revision sheet: two quick passes

Pass 1: remember the story in 12 sentences

1. A robot is a programmable physical system that performs tasks using mechanical action.
2. A typical controlled robot combines links, joints, actuators, sensors, controller and an end-effector.
3. Cartesian robots use prismatic axes; articulated robots use rotary axes; SCARA often combines rotary and vertical motion.
4. DoF counts independent variables describing configuration, not the number of pieces you can see.
5. A free rigid body has 6 spatial pose freedoms; a planar rigid body has 3.
6. A coordinate frame is an origin plus right-handed orthogonal axes; every reported point must identify its frame.
7. A rotation matrix contains direction information without translation; $R^T R = I$ and $\det R = 1$.
8. Rotation order matters: matrix products usually do not commute.
9. A homogeneous transform stores orientation R and origin shift t in a 4×4 matrix.
10. Frame chains multiply, and reversing a transform requires R^T and $-R^T t$.
11. Euler ZYZ is $R_z(\alpha)R_y(\beta)R_z(\gamma)$ and has angle ambiguity when $\beta = 0$ or π.
12. RPY ZYX is $R_z(\psi)R_y(\theta)R_x(\phi)$ and has angle ambiguity when pitch is $\pm 90^\circ$.

Pass 2: write a five-minute exam response

Pick one definition or derivation from the bank. Start with a clear one-line definition; add the physical meaning and diagram; derive or state the required formula with correctly defined symbols; carry out one checked example if needed; end with an application or limitation. **Do not write formula lists without explaining the variables.**

Rapid diagram recall

Without looking, draw (a) sensor-controller-actuator-feedback loop; (b) a four-bar with ground and joints marked; (c) two offset frames and point P; (d) a positive planar rotation; (e) a 4-by-4 homogeneous transform. Compare with the diagrams in the body of this guide.

13 Formula and convention sheet

A. Degrees of freedom

$$\boxed{M_{\text{planar}} = 3(n-1) - 2j_1 - j_2} \qquad \boxed{M_{\text{spatial}} = 6(n-1-j) + \sum f_i}$$

n includes ground; j_1 is number of 1-DoF planar lower pairs; j_2 is typical planar higher pairs; spatial formula assumes independent generic constraints.

B. Planar and 3D basic rotations

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha \\ 0 & s_\alpha & c_\alpha \end{bmatrix}, \qquad R_y(\beta) = \begin{bmatrix} c_\beta & 0 & s_\beta \\ 0 & 1 & 0 \\ -s_\beta & 0 & c_\beta \end{bmatrix},$$

$$R_z(\gamma) = \begin{bmatrix} c_\gamma & -s_\gamma & 0 \\ s_\gamma & c_\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad c_\eta = \cos\eta,\ s_\eta = \sin\eta.$$

$$R^T R = I, \quad R^{-1} = R^T, \quad \det R = 1, \quad \|Rp\| = \|p\|.$$

C. Homogeneous pose and conversion

$$\boxed{{}^A T_B = \begin{bmatrix} {}^A R_B & {}^A t_B \\ 0\ 0\ 0 & 1 \end{bmatrix}}, \qquad \boxed{{}^A p = {}^A R_B\, {}^B p + {}^A t_B}$$

$$\boxed{{}^B T_A = ({}^A T_B)^{-1} = \begin{bmatrix} R^T & -R^T t \\ 0\ 0\ 0 & 1 \end{bmatrix}}, \qquad {}^A T_C = {}^A T_B\, {}^B T_C.$$

Homogeneous point $w = 1$ gets rotated and translated; direction $w = 0$ gets only rotated.

D. Euler and roll-pitch-yaw conventions

$$\boxed{R_{ZYZ} = R_z(\alpha)R_y(\beta)R_z(\gamma)}$$

$$\boxed{R_{ZYX} = R_z(\psi)R_y(\theta)R_x(\phi)}$$

ZYZ ambiguity at $\beta = 0$ or π. ZYX ambiguity at $\theta = \pm 90°$. Always declare sequence, axes, rotation direction and units. atan2 is preferred for angle extraction.

Quality-control checklist for any numerical: sketch axes; choose a convention; write inputs and units; multiply in correct order; simplify; test a limit or special case; box the result with units.

14 Sources, verification and responsible use

Authoritative syllabus: Savitribai Phule Pune University, *TE Robotics and Automation Engineering (2024 Pattern)*, effective AY 2026-27, PCC303-ROA, printed p. 19, source PDF. Course and Unit I topics transcribed from that document.

Primary technical reference: Kevin M. Lynch and Frank C. Park, *Modern Robotics: Mechanics, Planning, and Control*, Cambridge University Press, 2017; authors' preprint, sections on configuration space and rigid-body motions. Official textbook/course website.

Additional syllabus-listed references: Robert J. Schilling, *Fundamentals of Robotics*; Mark W. Spong et al., *Robot Modeling and Control*; John J. Craig, *Introduction to Robotics: Mechanics and Control*.

Existing project material reviewed: *RKD Complete Exam Notes, Edition 3* (Zenith/Om Deshmukh), checked for topic coverage and presentation approach. This new unit is freshly worded, with new diagrams and recalculated examples.

Past-paper provenance: SPPU T.E. Automation and Robotics RKD, Nov 2025, paper 6583-177 is explicitly a *2019 Pattern* paper; it helps identify overlapping topics but does not establish 2024 Pattern question weighting.

Methodology: Original and extended worked examples; calculations checked independently by the accompanying Python verifier. Diagrams are original vector schematics prepared for Zenith, not copied screenshots. No claim is made that these exact practice questions will appear in an examination.

Editorial release policy: One official syllabus → one topic ledger → evidence per topic → original explanations and diagrams → numerical recomputation → PDF rendering QA → verified Drive publishing and website linking. Generated content is never treated as a substitute for the original syllabus or lecturer guidance.